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## Split Generation in the SUSY Mass Spectrum and $B_s - \bar{B}_s$ Mixing

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### Abstract

We show that the like-sign di-muon anomaly reported recently by the D0 Collaboration can be explained in the supersymmetric standard model (SM) if the squarks and the sleptons in the first two generations have relatively small, but degenerate mass spectrum, and those in the third generation are larger as  $O(1 - 10)\text{TeV}$ . This split generation model provides large contributions to the  $B_s - \bar{B}_s$  mixing, although most of the FCNC's are suppressed due to the large masses of the third generation squarks or the GIM mechanism partially acting on the first and second generations.

# 1 Introduction

Gravity mediation [1] is a well-motivated and simple mechanism for transmitting supersymmetry (SUSY) breaking to the observed sector. However, it predicts too much flavor-changing neutral currents (FCNC's) in a generic vacuum of the gravity mediation. The most popular solution to this FCNC problem is to postulate a degenerate SUSY-breaking mass spectrum for squarks and sleptons. However, the origin of such a mass degeneracy is not known. An alternative solution was considered where the masses of squarks and sleptons in the first and second generations are relatively large as  $O(10)\text{TeV}$  and the masses for the third generation are in the range of  $O(100)\text{GeV}-1\text{TeV}$ . This is called as “decoupling model” [2].

Very recently, a new solution to the FCNC problem has been proposed where the squarks and sleptons in the first and second generations have relatively small, but degenerate SUSY-breaking masses and the squarks and sleptons in the third generation are larger as  $O(1-10)\text{TeV}$ . This is motivated by the Nambu-Goldstone (NG) hypothesis for quarks and leptons in the first two generations [3]. In fact, the SUSY non-linear sigma model,  $E_7/SO(10) \times U(1)^2$ , is known [4, 5] to accommodate two generations of quark and lepton chiral multiplets as NG modes [6]. We call it “split generation model” to distinguish it from the decoupling model

We show, in this paper, that the split generation model may explain the like-sign dimuon anomaly observed recently in the D0 experiment [7], without generating any conflict with all experimental constraints. This is because it naturally predict a relatively large  $B_s - \bar{B}_s$  mixing with a new CP phase, while other FCNC processes are suppressed. In particular, it is stressed that the model can easily satisfy the bound from  $\text{Br}(b \rightarrow s\gamma)$ , in contrast to other two models with the degenerate or the decoupling mass spectra. This is the main reason why we propose the split generation model as a serious candidate for the beyond SM.

Before going to detailed analyses in this paper, it should be discussed here on the other problem in the gravity mediation, that is, the  $CP$  problem. The gravity mediation involves several  $CP$  violating phases in SUSY-breaking soft masses and it predicts too large  $CP$  violation at low energies. Since there has been found no solution to this problem so far,

we simply assume, throughout this paper, that  $CP$  is not broken in the SUSY breaking sector. Therefore, all  $CP$  violating phases originate only from the diagonalization of quark and lepton mass matrices. From the theoretical point of view, it is more interesting to impose  $CP$  invariance in the full theory, and the  $CP$ -violating phase in the CKM matrix appears as a result of some spontaneous  $CP$  violation. This assumption of  $CP$  invariance itself may give a deep clue to a solution of the strong  $CP$  problem, since the QCD vacuum angle  $\theta_{\text{QCD}}$  vanishes at the fundamental cut-off scale. However, the quark rotations for the diagonalization of their mass matrices, in general, shift the QCD  $\theta$  angle and hence we have the strong  $CP$  violation even if the original QCD vacuum is  $CP$  invariant. Therefore, the hypothesis of  $CP$  invariance for the full theory does not solve the strong  $CP$  problem at the first glance. However, if the quark mass matrices are hermitian, the shift of the  $\theta_{\text{QCD}}$  angle vanishes and we can maintain  $\theta_{\text{QCD}} = 0$  even after the diagonalization of quark mass matrices [8]. This assumption on quark mass matrices becomes also very important to suppress neutron and atomic electric dipole moments sufficiently, as shown in section 5.

## 2 Split Generation Model

We consider the split generation model, in which the squarks have the following spectrum of SUSY-breaking soft masses,

$$\tilde{m}_q = \text{diag} (m_{\tilde{q}}, m_{\tilde{q}}, M_{\tilde{q}}), \quad (1)$$

for the three generations. Here, “split generation” means that the squarks in the first two generations have relatively small and degenerate SUSY-breaking soft masses,  $m_{\tilde{q}}$ , and the masses in the third generation are relatively large,  $M_{\tilde{q}} > m_{\tilde{q}}$ . On the other hand, the Yukawa matrices of the quarks are assumed to be non-diagonal. We call this basis as the generation basis.<sup>1</sup>

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<sup>1</sup>Note that this does not define the generation axes uniquely, since the soft masses of the first and second generations are degenerate, and we can rotate their axes without changing the SUSY-breaking soft mass matrices (1). Here, we introduce (very) tiny violations of the mass degeneracy, though they are neglected hereafter, since they are irrelevant for the following discussion. They are needed only for defining the generation basis.

The model generally induces FCNC's and CP violations through the mixings with the third generation. In the super-CKM basis, where the SUSY-invariant Yukawa matrices of the quark chiral multiplets are diagonalized, not only quarks but also squarks are rotated from the generation basis, and hence mass matrices of squarks are no longer diagonal due to the non-degenerate third component of (1). Moreover, since the Yukawa matrices are generally complex, the squark mass matrices acquire complex phases in the super-CKM basis independent of the CKM phase.<sup>2</sup>

In order to analyze the (CP-violating) FCNC's quantitatively, it is convenient to use the hybrid basis rather than the super-CKM basis, where only the quark mass matrices are diagonalized, while the squarks are left unrotated. In the hybrid basis, the gaugino–quark–squark vertices are flavor non-diagonal. Actually, when we change the basis from the generation to the hybrid basis, the gluino vertices are transformed as

$$\mathcal{L} = -\sqrt{2}g_s\bar{q}_L\tilde{g}^{(a)}T^{(a)}\tilde{q}_L + \cdots = -\sqrt{2}g_s\bar{q}'_L(U_L)^\dagger\tilde{g}^{(a)}T^{(a)}\tilde{q}_L + \cdots, \quad (2)$$

where  $g_s$  is the QCD gauge coupling constant with the color index  $(a)$ , and the unitarity matrices  $(U_{L(R)})_{ij}$  are diagonalization matrices for the mass matrices of the left- (right-) handed quarks. Since the rotation matrices of the up- and down-type quarks provide the CKM matrix  $V$ , the mixing angles  $(U_{L,R})_{23}$  and  $(U_{L,R})_{32}$  are expected to be  $\sim V_{ts} \sim \lambda^2$  with the Wolfenstein parameter  $\lambda \simeq 0.2$ .

The squark couplings to the Higgs/Higgsino can also change flavors. In particular, the chirality mixing of the squark mass matrix,  $(\tilde{m}_{LR,RL}^q)^2$ , have flavor off-diagonal elements. In fact, it depends on the Yukawa coupling such as  $(\tilde{m}_{LR,RL}^d)_{ij}^2 \simeq -(Y_d)_{ij}v\mu\tan\beta$  for the down-type squarks, where  $Y_d$  is the down-type Yukawa coupling,  $v \simeq 174\text{GeV}$  the SM Higgs vacuum expectation value (VEV),  $\mu$  the Higgsino mass, and  $\tan\beta$  a ratio of the up- and down-type Higgs VEVs. It is noticed that  $Y_q$  in  $(\tilde{m}_{LR,RL}^q)^2$  is non-diagonal in the hybrid basis and is represented as  $Y_q = U_L Y_q^{(\text{diag})} U_R^\dagger$ , where  $Y^{(\text{diag})}$  is the diagonalized Yukawa coupling. Similarly, the squark couplings to the neutralino/chargino have flavor-changing components, which come from the mixing between the neutralino/chargino and the Higgsino.

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<sup>2</sup> The CP violations are also represented by the Jarlskog invariants in SUSY [9].

### 3 $B_s - \bar{B}_s$ Oscillation

We first discuss the oscillation between the  $B_s^0$  and  $\bar{B}_s^0$  mesons. In fact, the like-sign di-muon asymmetry is sensitive to the oscillation. The D0 Collaboration recently reported the asymmetry [7],

$$A_{\text{sl}}^b \equiv \frac{N_b^{++} - N_b^{--}}{N_b^{++} + N_b^{--}} = -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3}, \quad (3)$$

where  $N_b^{++(-)}$  is the number of  $b\bar{b} \rightarrow \mu^{+(-)}\mu^{+(-)}X$  events. This result is  $3.2\sigma$  deviated from the SM prediction,  $A_{\text{sl}}^b = (-2.3_{-0.6}^{+0.5}) \times 10^{-4}$  [10]. At the Tevatron, it is related to the semileptonic CP asymmetries of the  $B_d$  and  $B_s$  mesons [7] as,

$$A_{\text{sl}}^s = (0.506 \pm 0.043)a_{\text{sl}}^d + (0.494 \pm 0.043)a_{\text{sl}}^s, \quad (4)$$

where  $a_{\text{sl}}^d$  and  $a_{\text{sl}}^s$  are given as

$$a_{\text{sl}}^q = \text{Im} \frac{\Gamma_{12}^q}{M_{12}^q} = \left| \frac{\Gamma_{12}^q}{M_{12}^q} \right| \sin \phi^q, \quad (5)$$

in terms of the off-diagonal elements of the mass and decay matrices,  $M_{12}^q$  and  $\Gamma_{12}^q$ , of the  $B_q - \bar{B}_q$  oscillation for  $q = d, s$ .

The oscillation contributes to other observables as well as the like-sign di-muon asymmetry (see [11] and [12]). In particular, the mass difference of the  $B_q$  mesons,  $\Delta m_q$ , provides a severe bound on the SUSY contributions. Additionally, the time-dependent  $B_s \rightarrow \psi\phi$  decay give a measurement of the width difference of the  $B_s$  mesons,  $\Delta\Gamma_s$ , and the time-dependent CP asymmetry,  $S_{\psi\phi}$ . Combining these observables, we obtain the  $\chi^2$  minimum at [11, 12] <sup>3</sup>

$$(h_s, \sigma_s) \simeq (0.5, 120^\circ) \text{ and } (1.8, 100^\circ), \quad (6)$$

for the  $B_s$  meson, where  $h_s$  and  $\sigma_s$  are defined by  $M_{12}^s = (M_{12}^s)^{\text{SM}}(1 + h_s e^{2i\sigma_s})$ . Actually, this significantly contributes to the phase,  $\phi^q$ , away from the SM value in order to explain the like-sign di-muon anomaly. On the other hand, the contribution to the  $B_d$  meson is constrained to be small.

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<sup>3</sup> After the analysis in [11, 12], the D0 Collaboration updated the result of  $B_s \rightarrow \psi\phi$  [13]. Although it turned to be SM consistent, the minimum positions are expected to be unaffected so much.

The split generation model dominantly contributes to the  $B_s - \bar{B}_s$  oscillation as is shown below. The oscillation is represented by the following  $\Delta B = 2$  effective Hamiltonian,

$$H_{\text{eff}} = \sum_{i=1}^5 C_i O_i + \sum_{i=1}^3 \tilde{C}_i \tilde{O}_i, \quad (7)$$

where the operators are

$$\begin{aligned} O_1 &= (\bar{s}_L^\alpha \gamma_\mu b_L^\alpha)(\bar{s}_L^\beta \gamma^\mu b_L^\beta), \\ O_2 &= (\bar{s}_R^\alpha b_L^\alpha)(\bar{s}_R^\beta b_L^\beta), \quad O_3 = (\bar{s}_R^\alpha b_L^\beta)(\bar{s}_R^\beta b_L^\alpha), \\ O_4 &= (\bar{s}_R^\alpha b_L^\alpha)(\bar{s}_L^\beta b_R^\beta), \quad O_5 = (\bar{s}_R^\alpha b_L^\beta)(\bar{s}_L^\beta b_R^\alpha), \end{aligned} \quad (8)$$

and  $\tilde{O}_i$  by  $R \leftrightarrow L$ . In the decoupling limit,  $M_{\tilde{q}} \gg m_{\tilde{q}}$ , the SUSY contributions to the Wilson coefficients become

$$C_1 = -\frac{\alpha_s^2}{216} \frac{1}{m_{\tilde{q}}^2} [24xf(x) + 66\tilde{f}(x)] (U_L)_{23}^2, \quad (9)$$

$$C_2 = -\frac{\alpha_s^2}{216} \frac{1}{m_{\tilde{q}}^2} 204xf(x)(\delta_{RL})_{23}^2, \quad C_3 = \frac{\alpha_s^2}{216} \frac{1}{m_{\tilde{q}}^2} 36xf(x)(\delta_{RL})_{23}^2, \quad (10)$$

$$C_4 = -\frac{\alpha_s^2}{216} \frac{1}{m_{\tilde{q}}^2} [(504xf(x) - 72\tilde{f}(x))(U_L)_{23}(U_R)_{23} - 132\tilde{f}(x)(\delta_{LR})_{23}(\delta_{RL})_{23}], \quad (11)$$

$$C_5 = -\frac{\alpha_s^2}{216} \frac{1}{m_{\tilde{q}}^2} [(24xf(x) + 120\tilde{f}(x))(U_L)_{23}(U_R)_{23} - 180\tilde{f}(x)(\delta_{LR})_{23}(\delta_{RL})_{23}], \quad (12)$$

$$\tilde{C}_1 = -\frac{\alpha_s^2}{216} \frac{1}{m_{\tilde{q}}^2} [24xf(x) + 66\tilde{f}(x)] (U_R)_{23}^2, \quad (13)$$

$$\tilde{C}_2 = -\frac{\alpha_s^2}{216} \frac{1}{m_{\tilde{q}}^2} 204xf(x)(\delta_{LR})_{23}^2, \quad \tilde{C}_3 = \frac{\alpha_s^2}{216} \frac{1}{m_{\tilde{q}}^2} 36xf(x)(\delta_{LR})_{23}^2, \quad (14)$$

where the LR and RL mixing parameters are defined as  $(\delta_{LR,RL})_{ij} \equiv (\tilde{m}_{LR,RL}^d)_{ij}^2/M_{\tilde{q}}^2$ , and the loop functions are

$$f(x) = \frac{2(1-x) + (1+x)\ln x}{(x-1)^3}, \quad \tilde{f}(x) = \frac{1-x^2 + 2x\ln x}{(x-1)^3}, \quad (15)$$

with a ratio of the gluino mass to the light squark mass,  $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ . In the expression, the terms with  $(U_L)_{23}$  and  $(U_R)_{23}$  come from the strange squark diagrams, while the bottom-squark contributions with  $(U_L)_{23}$  and  $(U_R)_{23}$  are discarded, since they are suppressed by

a heavy squark mass  $M_{\tilde{q}}$ . On the other hand, the contributions with the chirality flip,  $(\delta_{LR,RL})_{ij}$ , are always suppressed by  $M_{\tilde{q}}$ , because they inevitably depend on the scalar-bottom quark.

From the above effective Hamiltonian, we obtain the dispersive part of the  $B_s - \bar{B}_s$  mixing amplitude,  $M_{12}^s \equiv \langle B_s^0 | H_{\text{eff}} | \bar{B}_s^0 \rangle$ , in terms of the hadron matrix elements,  $\langle B_s^0 | O_i | \bar{B}_s^0 \rangle$ . Numerically, it becomes

$$\begin{aligned} h_s e^{2i\sigma_s} &= \frac{(M_{12})^{\text{SUSY}}}{(M_{12})^{\text{SM}}} \\ &\simeq \left[ 66 \left[ (U_L)_{23}^2 + (U_R)_{23}^2 \right] - 1.4 \times 10^3 (U_L)_{23} (U_R)_{23} \right. \\ &\quad \left. + 2.1 \times 10^2 \left[ (\delta_{RL})_{23}^2 + (\delta_{LR})_{23}^2 \right] - 1.0 \times 10^3 (\delta_{LR})_{23} (\delta_{RL})_{23} \right] \left( \frac{m_{\tilde{q}}}{400 \text{ GeV}} \right)^{-2}, \end{aligned} \quad (16)$$

for  $m_{\tilde{q}} = m_{\tilde{g}}$ . We find that the oscillation is enhanced when both  $(U_L)_{23}$  and  $(U_R)_{23}$  are large. Then, the CP symmetry is violated by their phases. Although a coefficient of the term depends on the mass spectrum, it can be checked that it is less sensitive to  $x$ . For instance, it becomes  $1.4(1.0) \times 10^3$  for  $m_{\tilde{g}} = 400(1000) \text{ GeV}$  and  $m_{\tilde{q}} = 400 \text{ GeV}$ . We also obtain the similar result for the  $B_d - \bar{B}_d$  oscillation, but the mixing is provided by  $(U_{L,R})_{13}$  and  $(\delta_{LR,RL})_{13}$ . Thus, it is suppressed.

From (16), we estimate the SUSY contributions to the  $B_s - \bar{B}_s$  oscillation. Numerically, we find that they become as large as the value which is required to explain the di-muon anomaly (6). Actually, postulating  $|(U_L)_{23}| = |(U_R)_{23}| = \lambda^2$  with  $\lambda \simeq 0.2$ , we obtain  $h_s = 1 - 2$  and  $\simeq 0.5$  for  $m_{\tilde{q}} = 420 - 600 \text{ GeV}$  and  $\simeq 900 \text{ GeV}$ , respectively. On the other hand, the anomalous like-sign di-muon charge asymmetry requires a large CP violation in  $B_s - \bar{B}_s$ . When the mixings are  $|(U_L)_{23}| \simeq |(U_R)_{23}|$ ,  $\sigma_s$  is related to the phase of  $(U_L)_{23}$  and  $(U_R)_{23}$  as

$$\Theta^+ \equiv \arg[(U_L)_{23}(U_R)_{23}] \simeq 2\sigma_s - 180^\circ. \quad (17)$$

Thus, from (6) the phases are required to be  $\Theta^+ \simeq 60^\circ$  and  $20^\circ$  for  $h_s \simeq 0.5$  and  $1.8$ , respectively.

When masses of the third generation squarks are heavy, the  $(\delta_{LR,RL})$  contributions are very small, since  $(\delta_{LR,RL})$  are suppressed by  $1/M_{\tilde{q}}^2$ . Actually, taking a typical value for the Yukawa coupling,  $(Y_d)_{23} \simeq (0.01 - 0.1)$ , we find that they are too small to satisfy (6) for  $M_{\tilde{q}} \gtrsim 1 \text{ TeV}$ . Thus, we will discard them in the following.

There are two comments on the SUSY contributions to the like-sign di-muon anomaly. In the split generation model, the SUSY contribution to the  $B_d$  oscillation is suppressed. This is because the mixings are expected to be  $|(U_L)_{13}| \simeq |(U_R)_{13}| \simeq 10^{-(2-3)}$  from the CKM matrix. We obtain  $h_d \lesssim 0.1$  even for  $m_{\tilde{q}} = 420 - 600 \text{ GeV}$ . Since the contribution is small, it is difficult to explain the di-muon anomaly solely by the  $B_d$  mixing, rather this can improve  $\chi^2$  with a proper phase when the  $B_s$  contribution is large [11, 12]. Also, the di-muon anomaly favors  $\Gamma_{12}^s$  to be deviated from the SM prediction, although uncertainties are still large [11, 12]. However, the split generation model does not change  $\Gamma_{12}^s$  from the SM value.

As a final remark in this section, we discuss the experimental constraint from the  $K - \bar{K}$  mixing. The SUSY contribution is obtained by substituting  $[(U_{L,R})_{23}(U_{L,R}^*)_{13}]^2$  for  $(U_{L,R})_{23}^2$  in Eq. (9)–(14). This is because the leading contributions which depend on  $(U_{L,R})_{12}$  almost cancel out due to the GIM mechanism. However, the light down and strange squarks contribute to the  $K - \bar{K}$  mixing through the term of  $[(U_{L,R})_{23}(U_{L,R}^*)_{13}]$  (due to the unitarity condition  $\sum_i (U_{L(R)})_{2i}(U_{L(R)}^*)_{1i} = 0$ ). It is remarkable that the experimental constraints from  $\Delta m_K$  and  $\epsilon_K$  are avoided for  $m_{\tilde{q}} \gtrsim 400 \text{ GeV}$  with  $|(U_{L,R})_{23}| \sim 10^{-2}$  and  $|(U_{L,R})_{13}| \sim 10^{-3}$ .

## 4 $\text{Br}(b \rightarrow s\gamma)$

The SUSY contribution to the  $b - s$  transition is constrained from the inclusive  $b \rightarrow s\gamma$  decay. In fact, the experimental result [14] agrees well with the SM prediction [15], restricting the extra contribution apart from the SM in the range,

$$-3 \times 10^{-5} < \Delta \text{Br}(b \rightarrow s\gamma) < 1.4 \times 10^{-4} \quad (18)$$

at the  $2\sigma$  level.

The  $\Delta B = 1$  effective Hamiltonian relevant for  $\text{Br}(b \rightarrow s\gamma)$  is

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts} \sum_{i=7\gamma, 8G} C_i O_i + \tilde{C}_i \tilde{O}_i, \quad (19)$$

where the operators are

$$O_{7\gamma} = \frac{e}{16\pi^2} m_b \bar{s}_L (F \cdot \sigma) b_R, \quad O_{8G} = \frac{g_s}{16\pi^2} m_b \bar{s}_L (G \cdot \sigma) b_R, \quad (20)$$



and  $R \leftrightarrow L$  for  $\tilde{O}_i$ . The SUSY contribution to  $C_{7\gamma}$  and  $C_{8G}$  are dominated by the gluino diagrams. Furthermore, since the external quarks have opposite chirality to each others, the contribution is enhanced by  $\tan\beta$  when the chirality flip takes place on the down-type squark propagator (see Fig. 1). In the hybrid basis, the flavor changing couplings appear in the gaugino vertex and the chirality flip of the squark, and the Wilson coefficients become

$$\begin{aligned} C_{7\gamma} &= -\frac{\sqrt{2}\alpha_s\pi}{6G_F V_{tb}V_{ts}} \sum_{i,j} \frac{(m_d)_{ji}}{m_b} \frac{m_{\tilde{g}}\mu \tan\beta}{m_{\tilde{q}i}^2 m_{\tilde{q}j}^2} (U_R)_{i3} (U_L^*)_{j2} \left[ -\frac{4}{3} M_1(x_i, x_j) \right], \\ C_{8G} &= -\frac{\sqrt{2}\alpha_s\pi}{2G_F V_{tb}V_{ts}} \sum_{i,j} \frac{(m_d)_{ji}}{m_b} \frac{m_{\tilde{g}}\mu \tan\beta}{m_{\tilde{q}i}^2 m_{\tilde{q}j}^2} (U_R)_{i3} (U_L^*)_{j2} \left[ -\frac{1}{6} M_1(x_i, x_j) + \frac{3}{2} M_2(x_i, x_j) \right], \end{aligned} \quad (21)$$

where  $(m_d)_{ij}$  is defined as  $(m_d)_{ij} = (Y_d)_{ij}v = (U_L Y_d^{(\text{diag})} U_R^\dagger)_{ij}v$ . The loop functions are

$$\begin{aligned} M_1(x, y) &= \frac{x^2(1-3y) - y^2(1-3x) + x - y}{(x-1)^2(y-1)^2(x-y)} - 2 \frac{x^2(y-1)^3 \ln x - y^2(x-1)^3 \ln y}{(x-1)^3(y-1)^3(x-y)}, \\ M_2(x, y) &= \frac{x^2(1+y) - y^2(1+x) - 3(x-y)}{(x-1)^2(y-1)^2(x-y)} + 2 \frac{x(y-1)^3 \ln x - y(x-1)^3 \ln y}{(x-1)^3(y-1)^3(x-y)}, \end{aligned}$$

with  $x_i = m_{\tilde{g}}^2/m_{\tilde{q}i}^2$ . Although there are additional contributions from chargino, neutralino and charged Higgs, we discard them for simplicity.

We notice that the split generation model easily satisfies the  $b \rightarrow s\gamma$  bound in the mass region which is indicated by the di-muon anomaly. This is because the bottom-squark contribution is suppressed by a large mass,  $M_{\tilde{q}}$ . The strange-squark diagram is independent of  $M_{\tilde{q}}$ , but the contribution is small because the chirality flip is given by  $m_s$ . Consequently, the SUSY contribution to  $b \rightarrow s\gamma$  is estimated to be  $< O(10^{-5})$  for  $m_{\tilde{q}} \sim 100\text{GeV}$  and  $M_{\tilde{q}} \sim 1\text{TeV}$  with  $\tan\beta = 10$  and  $|(U_L)_{23}| = |(U_R)_{23}| = \lambda^2$ . This result is much less than the experimental bound (18). Furthermore, although a large CP violation in the  $b-s$  transition can affect the time-dependent CP asymmetry of the  $B_d \rightarrow \phi K$  and  $\eta' K$  decays, a contribution from the chromo dipole operator, which usually dominates the SUSY contributions (see e.g. [16]), is suppressed by  $M_{\tilde{q}}$ , too.

The above result is contrasted to other mass models. The SUSY contributions to  $b \rightarrow s\gamma$  is generally large in the degenerate and decoupling mass models, since the third generation is relatively light. In fact, the bottom-squark exchange diagram dominates the SUSY contributions. Compared with the dominant contribution in the split generation

model, the bottom-squark contribution is enhanced by  $m_b/m_s$  due to a large chirality flip. On the other hand, the  $B_s - \bar{B}_s$  oscillation receives almost the same contribution as (16) from the bottom squark. Thus, they almost saturates or partially exceeds the experimental constraint. In addition, the time-dependent CP asymmetries of  $B_d \rightarrow \phi K$  and  $\eta' K$  are likely to receive a large contribution from the bottom-squark diagrams, which may exceed the experimental values. The above difficulty is also noted in the analysis on the di-muon anomaly and  $b \rightarrow s\gamma$  in [17] for the degenerate case, where the squark masses are taken quasi universal with respect to the generations,  $\tilde{m}_q \sim \text{diag}(m_{\tilde{q}}, m_{\tilde{q}}, m_{\tilde{q}})$ . We also obtain a very similar result for the decoupling model, since the bottom-squark contributions are not suppressed.

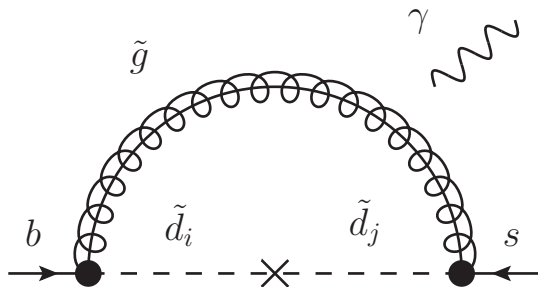


Figure 1: The gluino contribution to  $b \rightarrow s\gamma$ . The cross in the squark propagator denotes the chirality flip. The gluino vertex and the chirality flip of the squark have flavor structures.

## 5 Electric Dipole Moments

The anomalous like-sign di-muon charge asymmetry implies that the CP symmetry is violated in the squark sector. However, the violations are tightly limited by the EDM experiments. We assume throughout the present paper that all CP violating phases come only from the mixing matrices for quarks and leptons, as discussed in the Introduction. We see that even if they appear from the quark mixings, the electric and chromo-electric dipole moments (EDM's and CEDM's) can receive large contributions from superparticle

exchanges. The CEDM operators are defined by the effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = - \sum_f \frac{i}{2} \tilde{d}_f g_s \bar{\psi}_f (G \cdot \sigma) \gamma_5 \psi_f, \quad (22)$$

and the EDM operators are given similarly. In particular, according to [18], the CEDM of the strange quark is severely constrained by the neutron EDM,  $e|\tilde{d}_s| \lesssim 1 \times 10^{-25} \text{ ecm}$ , with updating the experimental data [19]. Since the estimation relies on the hadronic calculation and potentially includes a large uncertainty, we take a more conservative bound,  $e|\tilde{d}_s| < O(10^{-(24-25)}) \text{ ecm}$ , in the following study.

Through the 2 – 3 generation mixings, the Yukawa coupling receives a large contribution with the bottom quark mass,  $(Y_d)_{22} \supset (Y_d^{(\text{diag})})_{33} (U_L)_{23} (U_R^*)_{23}$ . Since the mixings  $(U_L)_{23}$  and  $(U_R)_{23}$  are complex in general, we obtain the SUSY contribution to the CEDM from the strange-squark loop as (c.f. [18])

$$\tilde{d}_s = \frac{\alpha_s}{4\pi} \frac{m_b m_{\tilde{g}} \mu \tan \beta}{m_{\tilde{q}}^4} |(U_L)_{23} (U_R^*)_{23}| \sin \Theta^- \left( -\frac{1}{6} M_1(x) + \frac{3}{2} M_2(x) \right), \quad (23)$$

where the CP phase is

$$\Theta^- \equiv \arg[(U_L)_{23} (U_R^*)_{23}]. \quad (24)$$

It is noticed that  $\Theta^-$  is different from  $\Theta^+$  in (17). Also, we find that the contribution is independent of the heavy squark mass,  $M_{\tilde{q}}$ . Here, the loop functions are defined as

$$M_1(x) = \frac{1 + 4x - 5x^2 + 2(2+x)x \ln x}{(x-1)^4}, \quad M_2(x) = \frac{5 - 4x - x^2 + 2(1+2x) \ln x}{(x-1)^4}, \quad (25)$$

with  $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ . The bottom squark gives the contribution as Eq. (23) with substituting the squark mass  $m_{\tilde{q}}$  by  $M_{\tilde{q}}$ , which is suppressed roughly by  $1/M_{\tilde{q}}^4$ .

The SUSY contribution (23) easily exceeds the experimental bound. We obtain a constraint on the CP phase  $\Theta^-$  as

$$|\sin \Theta^-| \lesssim 0.1(0.01) \left( \frac{\tan \beta}{10} \right)^{-1} \left( \frac{m_{\tilde{q}}}{500 \text{ GeV}} \right)^2, \quad (26)$$

from the experimental constraint  $e|\tilde{d}_s| < 10^{-24}(10^{-25}) \text{ ecm}$  for  $|(U_L)_{23}| = |(U_R)_{23}| = \lambda^2$  with assuming  $m_{\tilde{q}} = m_{\tilde{g}} = \mu$ . Thus, a phase of  $(U_L)_{23}$  is required to coincide with that of  $(U_R)_{23}$  at the 10(1)% accuracy.

In spite of the tight constraint on the phases, the CP phase of the  $B_s - \bar{B}_s$  oscillation can be large. This is because  $\Theta^-$  depends on  $(U_L)_{23}$  and  $(U_R)_{23}$  differently from  $\Theta^+$  which appears in  $B_s - \bar{B}_s$ . Namely, the  $B_s - \bar{B}_s$  mixing receives a contribution from  $\arg(U_L)_{23} + \arg(U_R)_{23}$ , while the EDM depends on  $\arg(U_L)_{23} - \arg(U_R)_{23}$ . Thus, an  $O(1)$  phase is allowed for  $(U_L)_{23}$  and  $(U_R)_{23}$ , as long as it satisfies  $\arg(U_L)_{23} \simeq \arg(U_R)_{23}$ . Such a condition is naturally realized when the Yukawa coupling is hermitian, since the rotation matrices become  $U_L = U_R$ , and the CEDM is suppressed,  $\Theta^- = 0$ . This hypothesis of the hermit Yukawa couplings may be very interesting in a connection with a solution to the strong CP problem [8], as discussed in the Introduction.

So far, we have discussed the case when both  $(U_L)_{23}$  and  $(U_R)_{23}$  are large. If either of them is suppressed, the situation gets worse. Since the SUSY contribution to the  $B_s - \bar{B}_s$  mixing dumps rapidly as is found from (16), we need a larger mixing angle to explain the di-muon anomaly such as  $|(U_L)_{23}|$  or  $|(U_R)_{23}| \sim 1$  for  $\bar{m}_{\tilde{q}} = 500\text{GeV}$ . Then, we obtain severer constraints from  $b \rightarrow s\gamma$  and/or the chargino or two-loop contributions to the neutron/atomic EDM's [20, 21].

## 6 Lepton Flavor Violations

Let us discuss flavor violations in the lepton sector. A generation mass splitting of the sleptons generally induces the lepton flavor violations (LFV's) through mixings between the leptons similarly to the squarks. In particular, if we consider the grand unification (GUT), the lepton sector is related to the quark sector, and we expect that the mixing angles of the lepton Yukawa couplings are also given by the CKM matrix. In this section, we analyze the flavor violating decay of the muon and the tau leptons in the hybrid flavor basis.

The severest constraint on the LFV's comes from the  $\mu \rightarrow e\gamma$  decay. From the experiments, the bound is known to be  $\text{Br}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$  at 90% [22]. The smuon-selectron diagram with the flavor-changing Yukawa coupling gives the branching ratio as

$$\text{Br}(\mu \rightarrow e\gamma) \simeq \frac{1}{\Gamma_{\text{tot}}} \frac{\alpha\alpha'^2}{1152\pi^2} m_\mu^3 m_\tau^2 \epsilon^2 \frac{M_1^2 \mu^2 \tan^2 \beta}{m_{\tilde{\ell}}^8}$$

$$\simeq 2 \times 10^{-12} \left( \frac{\epsilon}{10^{-5}} \right)^2 \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{m_{\tilde{\ell}}}{400 \text{GeV}} \right)^{-6} \left( \frac{\mu}{1 \text{TeV}} \right)^2, \quad (27)$$

in the limit of decoupled heavy sleptons,  $M_{\tilde{\ell}} \gg m_{\tilde{\ell}}$ . Here,  $\Gamma_{\text{tot}}$  is the total decay rate of the muon, and we assumed the Bino mass  $M_1$  is equal to the light slepton mass  $m_{\tilde{\ell}}$ , for simplicity. The mixing angle  $\epsilon$  is given by the lepton diagonalization matrices,  $U_{\ell_{L,R}}$ , such as  $(U_{\ell_L})_{23}(U_{\ell_R}^*)_{13}$  and  $(U_{\ell_R})_{23}(U_{\ell_L}^*)_{13}$ . Postulating  $(U_{\ell_L})_{23} \sim (U_{\ell_R})_{23} \sim 10^{-2}$  and  $(U_{\ell_L})_{13} \sim (U_{\ell_R})_{13} \sim 10^{-3}$  as is expected from the CKM matrix, the mixing angle is roughly  $\epsilon \sim 10^{-5}$ . Compared with the experimental result [22], we find that the split generation model satisfies the current constraint. Furthermore, it is in the reach of the sensitivity of the MEG experiment [23], as long as  $\mu \tan \beta$  is not very small.

Similarly to the  $b-s$  transition, the tau lepton can decay into the muon. The branching ratio of  $\tau \rightarrow \mu\gamma$  is bounded by the experiments,  $\text{Br}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8}$  at 90% [22]. Among the SUSY contributions, the smuon-chargino diagram is dominant, which is evaluated as

$$\begin{aligned} \text{Br}(\tau \rightarrow \mu\gamma) &\simeq \frac{1}{\Gamma_{\text{tot}}} \frac{\alpha \alpha_2^2}{144\pi^2} m_\tau^5 \frac{M_2^2 \tan^2 \beta}{\mu^2 m_{\tilde{\ell}}^4} |(U_{\ell_L})_{23}|^2 \\ &\simeq 1 \times 10^{-8} \left( \frac{|(U_{\ell_L})_{23}|}{\lambda^2} \right)^2 \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{m_{\tilde{\ell}}}{400 \text{GeV}} \right)^{-2} \left( \frac{\mu}{2 \text{TeV}} \right)^{-2}, \end{aligned} \quad (28)$$

in the decoupling limit, where  $\Gamma_{\text{tot}}$  means the total decay rate of the tau lepton, and  $M_2 = m_{\tilde{\ell}}$  is assumed in the second line. We notice that the rate is suppressed by a large  $\mu$  parameter, since the diagram includes the mixed Wino- and Higgsino-like charginos. Although the pure Wino contribution is not suppressed by  $M_{\tilde{\ell}}$ , this results in  $\text{Br}(\tau \rightarrow \mu\gamma) \simeq 10^{-11}$  for  $m_{\tilde{\ell}} = 400 \text{GeV}$ . As a result, we found that the SUSY contribution to  $\tau \rightarrow \mu\gamma$  easily satisfies the current experimental bound [22]. Furthermore, if the Higgsino is relatively light and  $\tan \beta$  is large, the branching ratio may be accessible in the super B factory [24].

## 7 Conclusions

In this letter, we have studied the split generation model, where the scalar fermions in the first and second generations have small and degenerate masses of  $O(100) \text{GeV}$ , and

those in the third generation are heavy with mass of  $O(1-10)\text{TeV}$ . The severe constraints from the FCNC's such as the  $K - \bar{K}$  mixing are suppressed sufficiently, while the mass difference between the squark generations generally induces the  $b - s$  transitions with a new CP phase. We have found that the SUSY contribution to the  $B_s - \bar{B}_s$  mixing can be large enough to explain the current anomaly of the like-sign di-muon charge asymmetry in the D0 experiment. We have also discussed that the contribution is consistent with the other experimental constraints such as  $b \rightarrow s\gamma$  and EDM's. This situation is contrasted to other mass models such as the degenerate or the decoupling models.

The model is also interesting from other view points of phenomenology. In addition to the di-muon anomaly, it generally predicts flavor violations in the lepton sector. It has been shown that the branching ratios of  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$  can be close to the sensitivities of the future experiments. Furthermore, the heavy third generation enhances light lepton events in the LHC experiment, which improves the SUSY detections and measurements [3]. Also, a light mass of the second generations allows a large contribution to the muon anomalous magnetic moment, which currently has a  $3-4\sigma$  discrepancy between the experimental data and the SM prediction [25]. Lastly, a heavy third generation is favored to satisfy the mass bound on the neutral Higgs boson from the LEP experiment [22]. We expect to test the split generation model in future experiments such as the LHC, LHCb, EDM's, superB factories, and MEG.

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